

# Long wave dynamics in the nearshore

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AUSTRALIA**

# Background

Long (infragravity) waves

Periods 20 s to 3 minutes.

Associated with short-wave groups.

Why?

Energetic in the nearshore.

Role in inundation and erosion.

# Background

## Types of long waves:

- Bound to the group -> propagating at  $C_g$ , in antiphase with the group envelope. There is a solution (Longuet-Higgins and Stewart, 1962), for constant depth and valid in deep enough water.
- Free -> detached from the groups.

## Two recognised mechanisms for generation of free long waves, associated with group breaking

- Breakpoint forcing (Symonds et al., 1982) -> steep slopes
- Bound long wave release (Masselink, 1995 ; Inch et al., 2017) -> mild slopes

Free waves generated over depths variations (Mei and Benmoussa, 1984; Janssen et al., 2003).

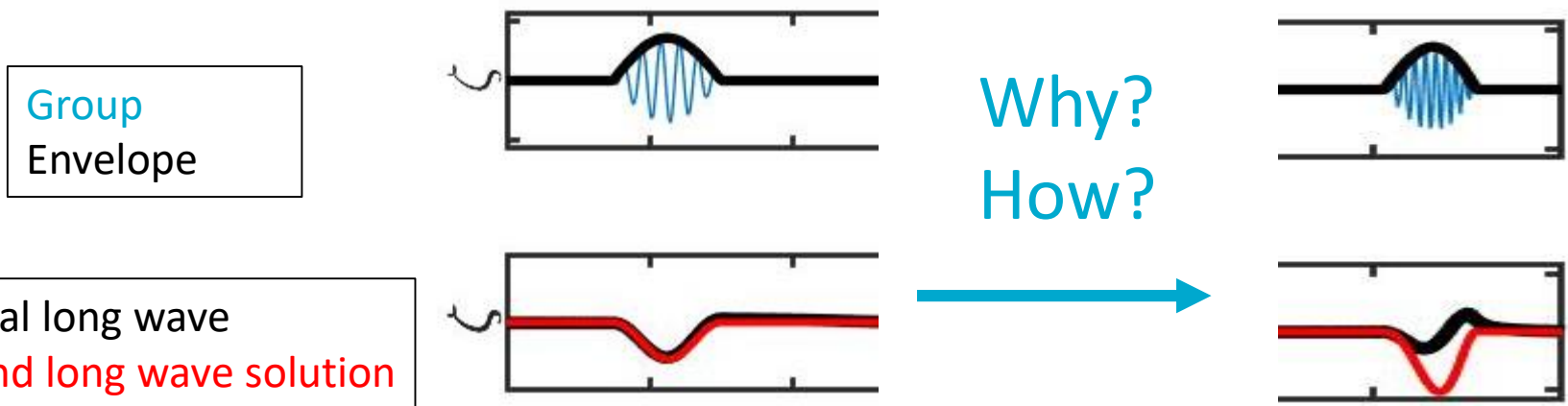
# Background

What we know (and what we don't know):

As wave groups propagate into shallow water,

- the phase, between the group envelope and the long shifts from  $180^\circ$  offshore to  $90^\circ$  approaching the shoreline (measured and modelled numerically).

- the bound wave solution (constant depth) overestimates the amplitude of the long waves.



# What this talk is about

- Understand the generation mechanisms
- Estimate phase lags and elevation amplitudes

We run a (simple) linear 1D numerical model

- isolate the mechanisms

# Some structure

- Simplest case, wave groups undisturbed (constant depth, no breaking)
- Most complicated case, depth varying and short-wave groups breaking

## Simplifications:

### Effect of depth variation

- Depth varying but no breaking
- One depth variation

### Effect of group breaking

- - Constant depth, breaking

# The important equations

Conservation of mass and momentum

Linearized, mild slope

Group scale

$$\frac{\partial \zeta}{\partial t} + h \frac{\partial U}{\partial x} = 0$$

$$\frac{\partial U}{\partial t} + g \frac{\partial \zeta}{\partial x} = - \frac{1}{\rho h} \frac{\partial S_{xx}}{\partial x}$$

Radiation stress gradient  
-> forcing for the model

Radiation stress

$$S_{xx} = \frac{1}{2} \rho g A_g^2 \left( \frac{2c_g}{c} - \frac{1}{2} \right),$$

Short-wave group envelope

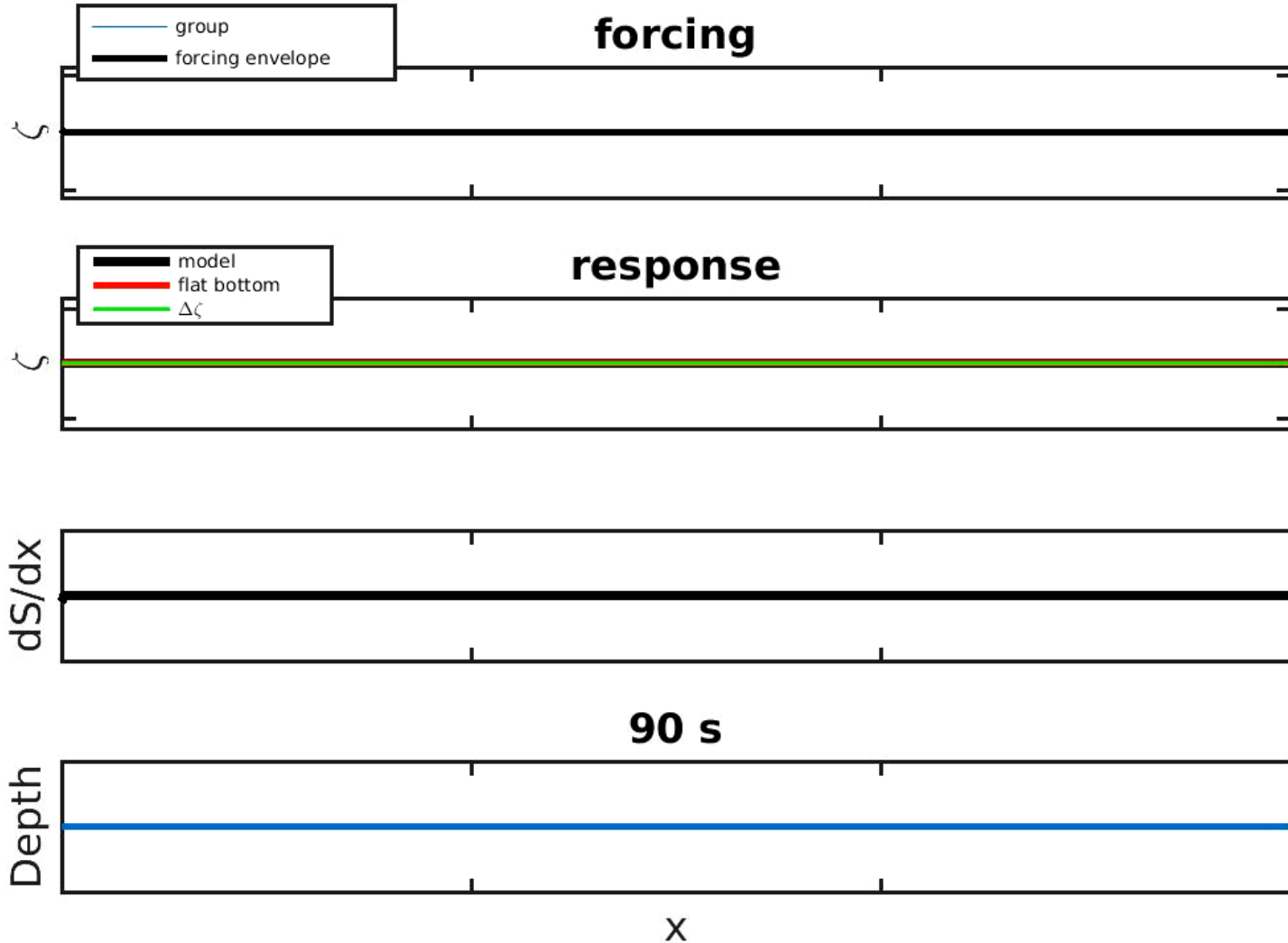
Depth

Flat bed solution  
(Longuet-Higgins and Stewart, 1962)

$$\zeta_{BLW} = - \frac{1}{\rho g h - c_g^2} S_{xx} + constant$$

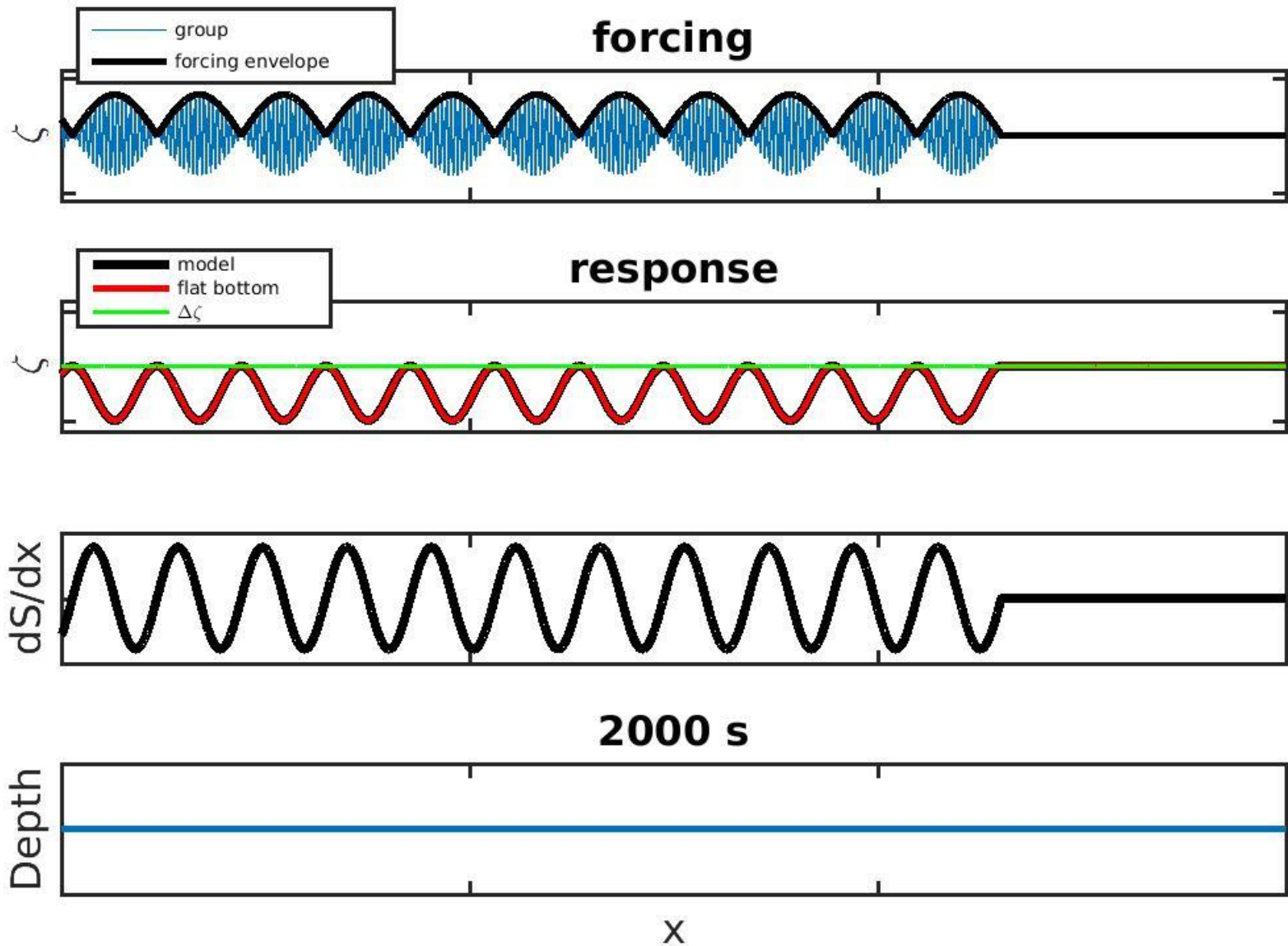
# Model run

## Flat



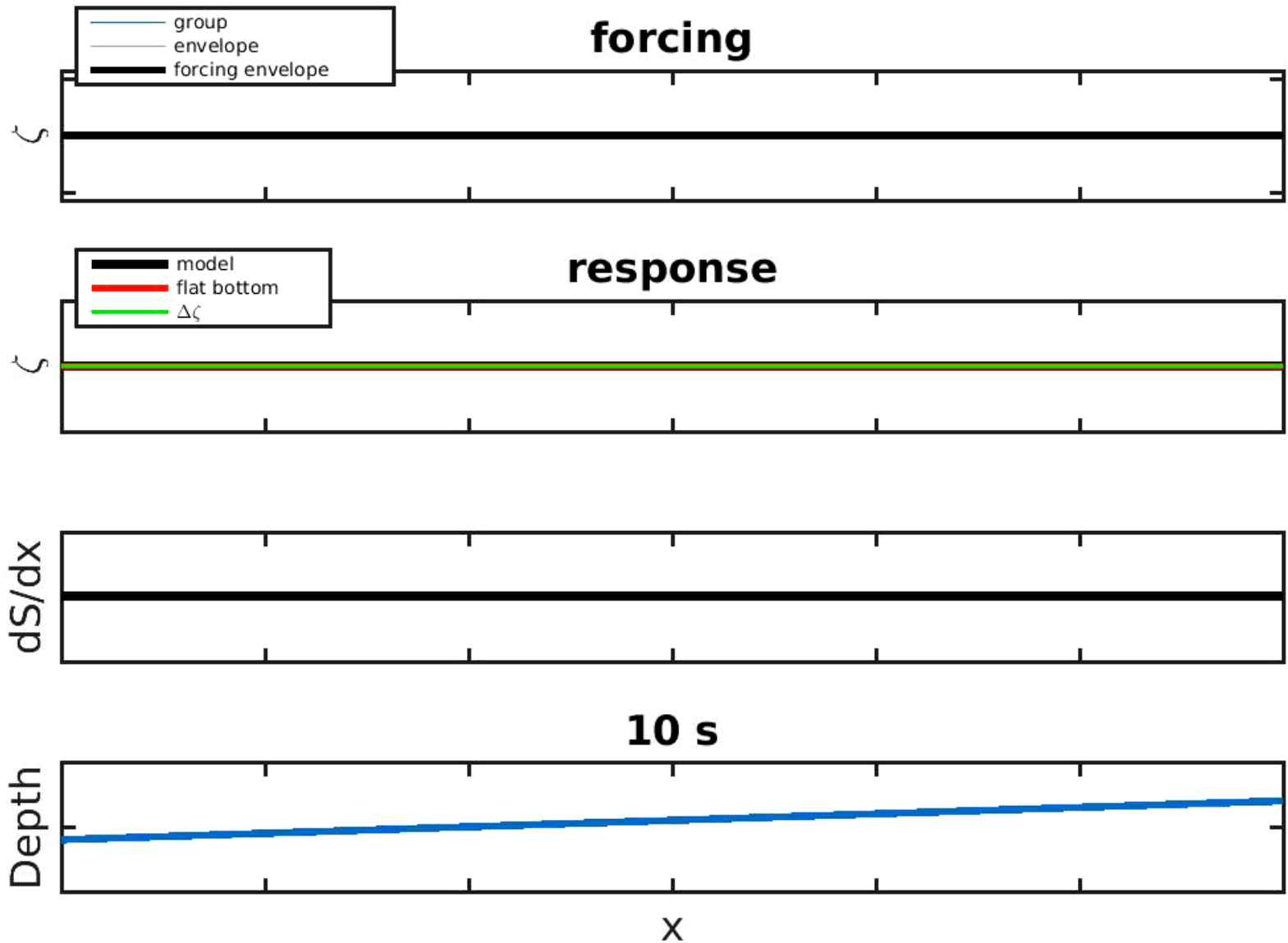


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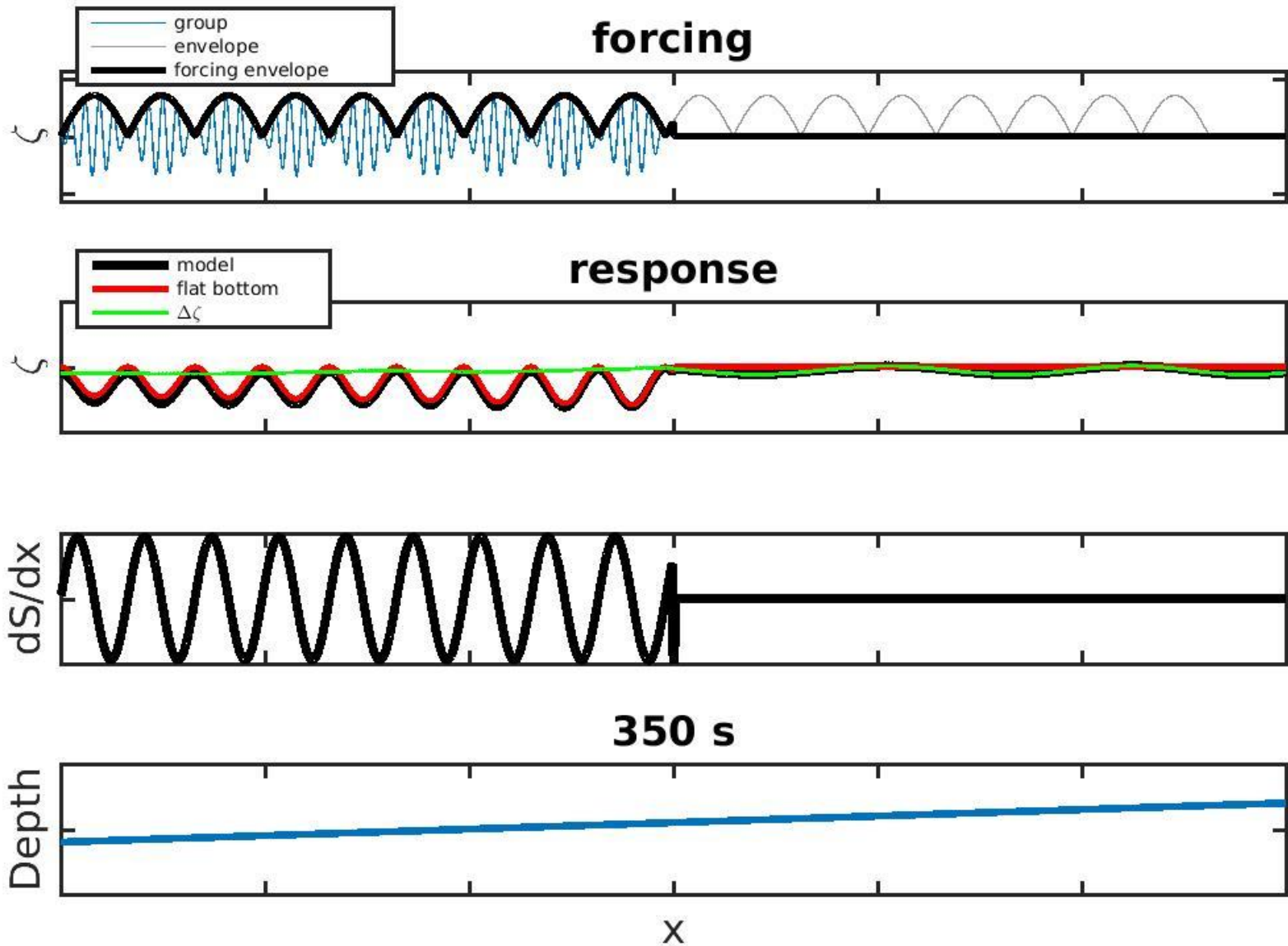


snapshot

# Model run



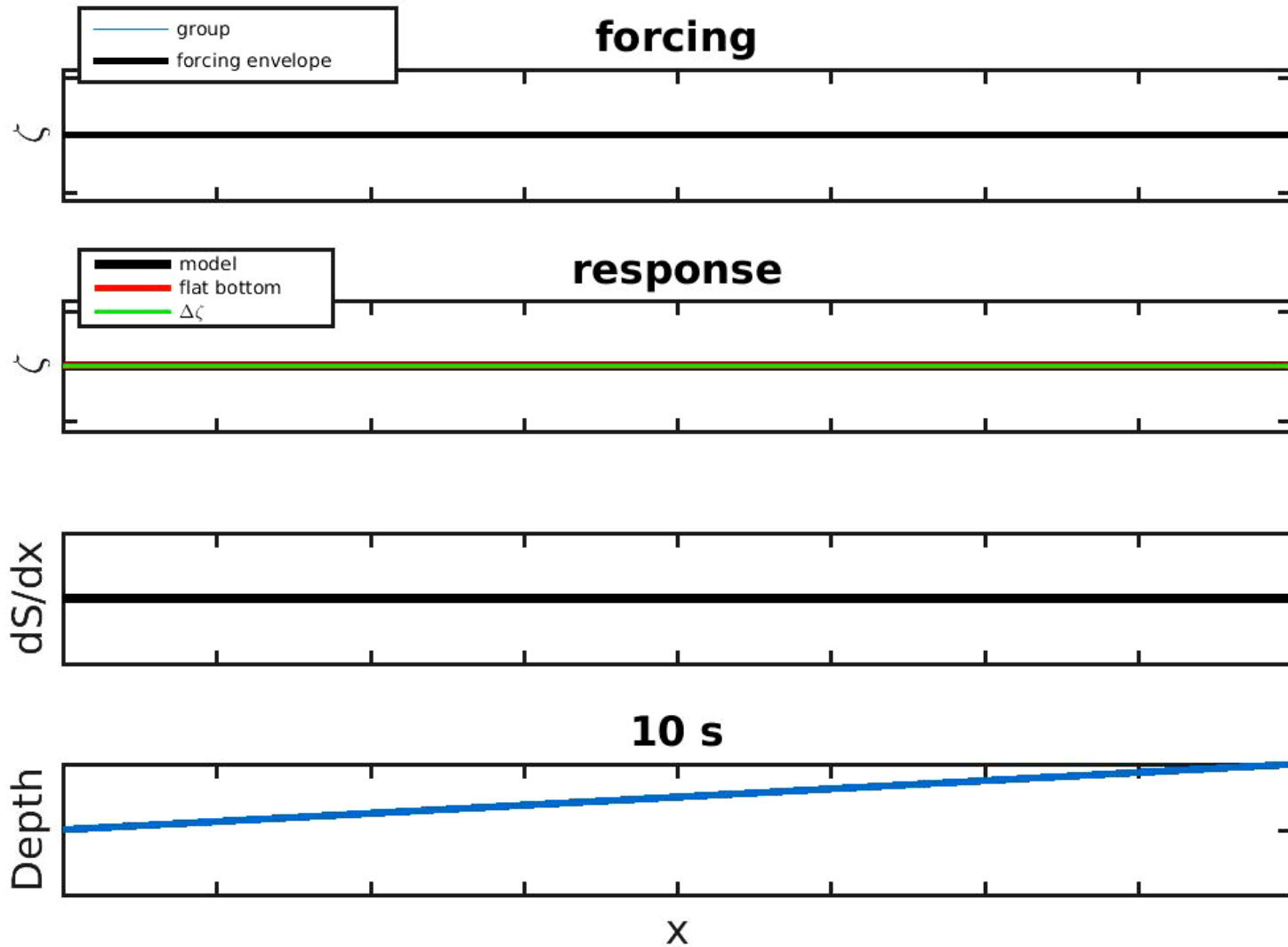
# Model run



snapshot

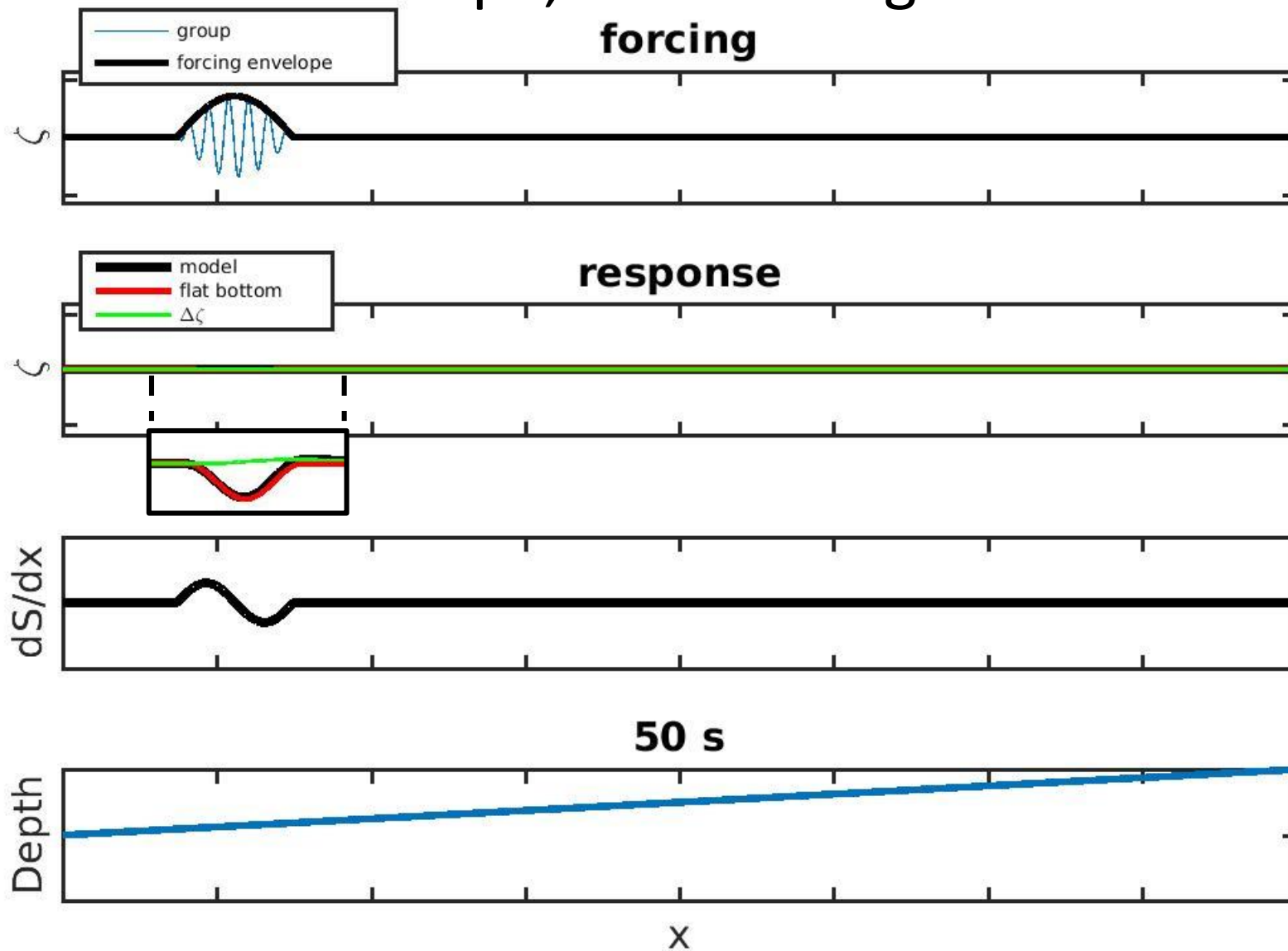
# Model run

## Slope, no breaking



# Model run

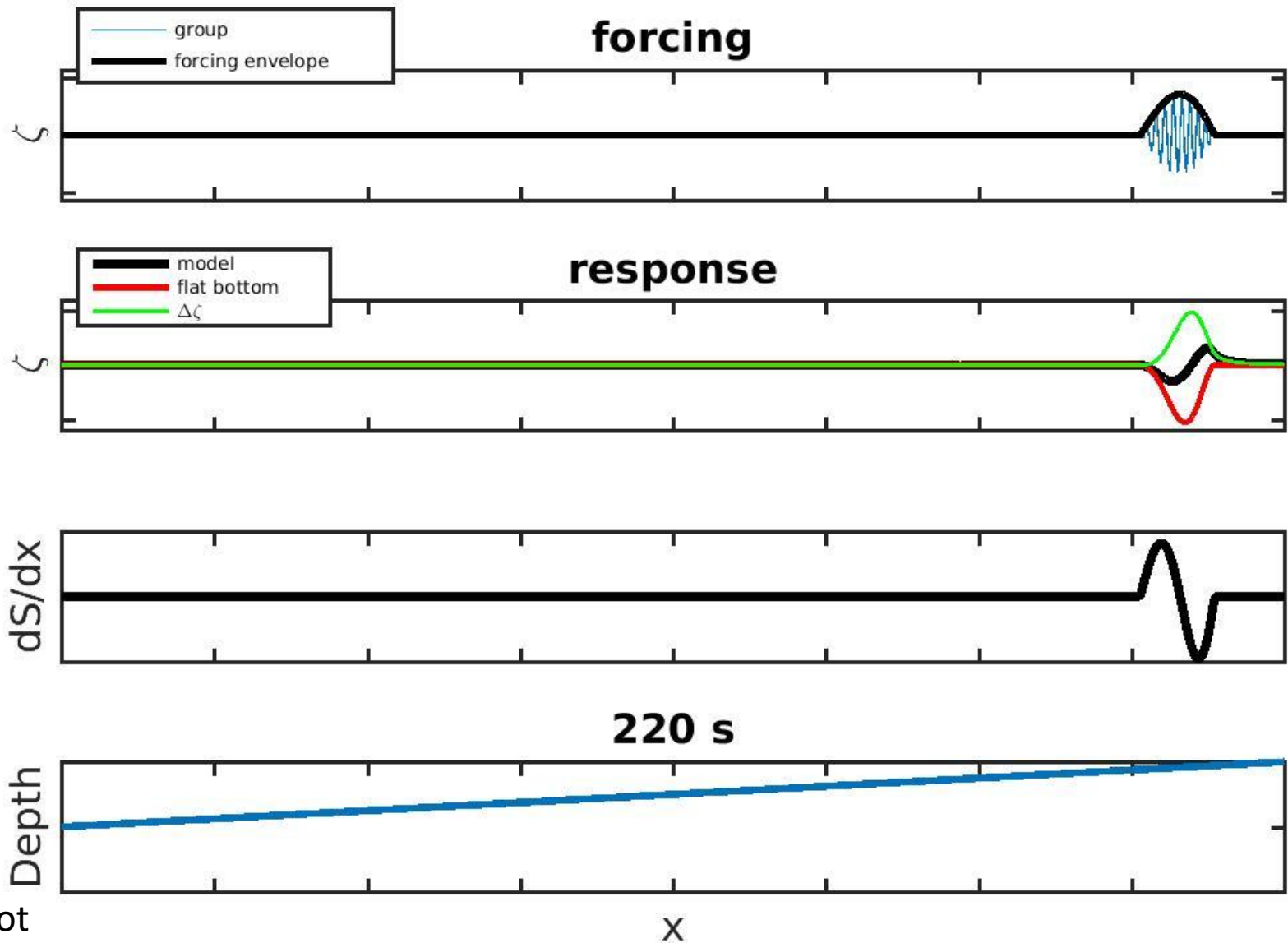
## Slope, no breaking



snapshot

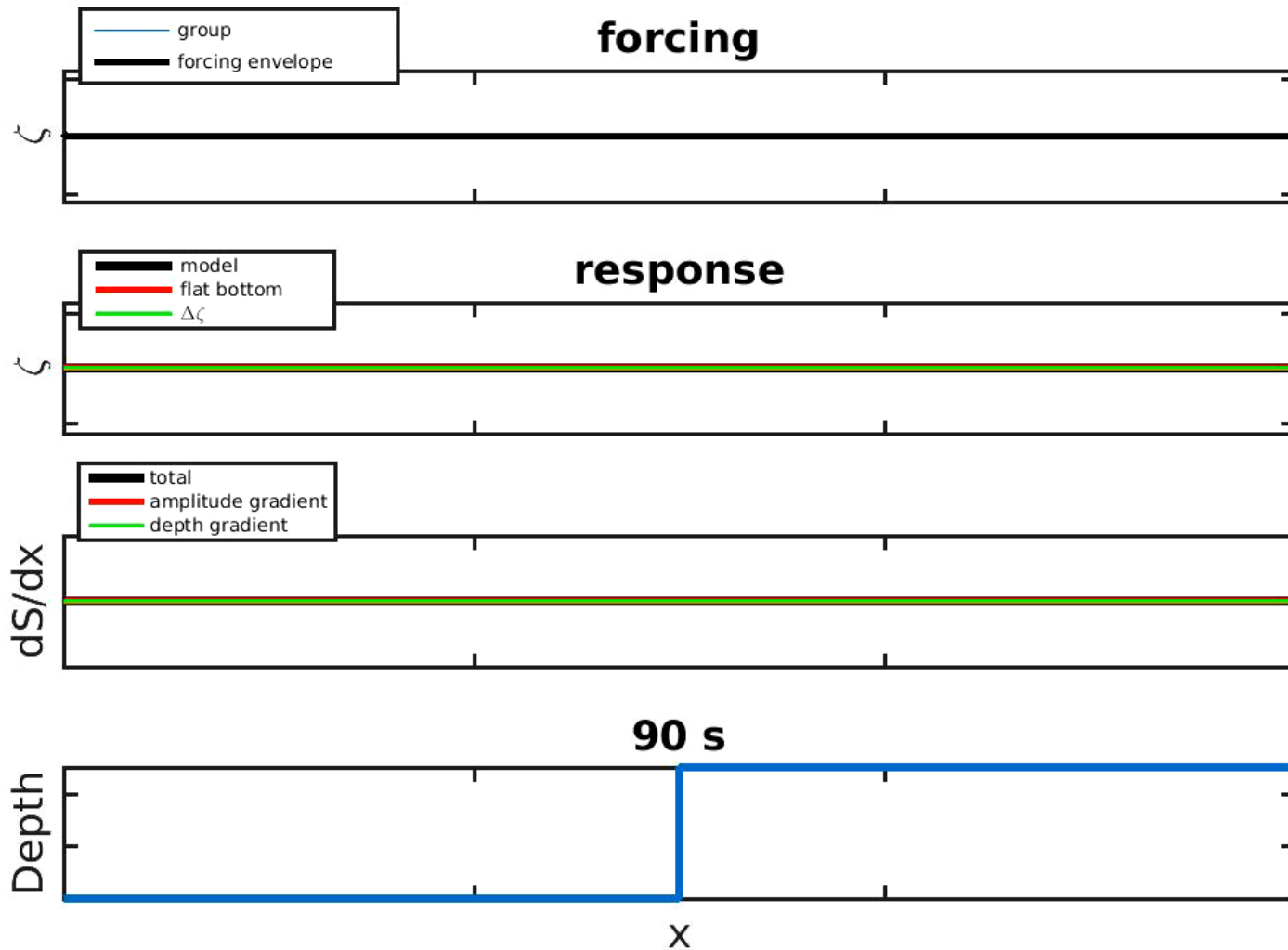
# Model run

## Slope, no breaking



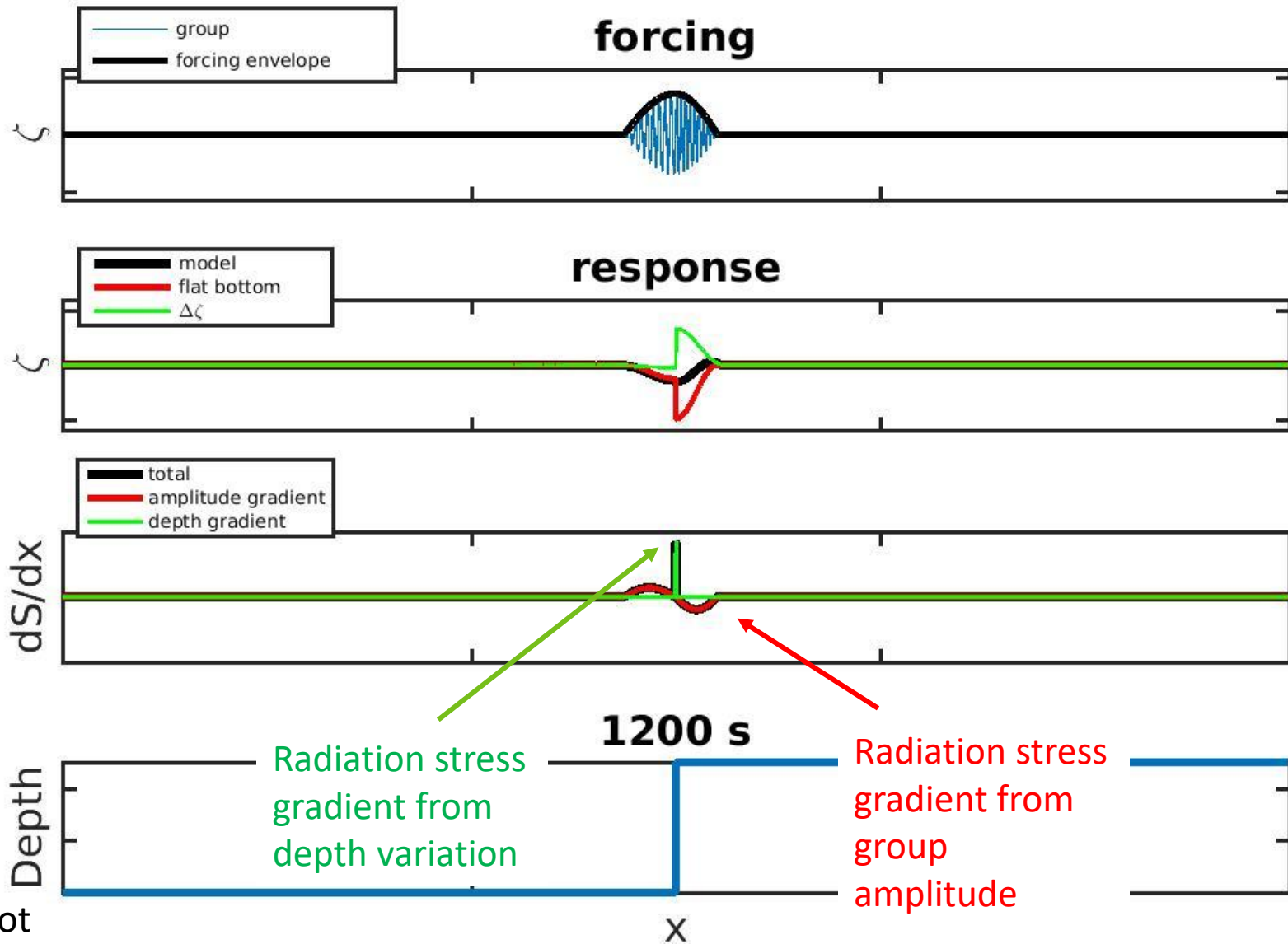
# Model run

## Single step



# Model run

## Single step



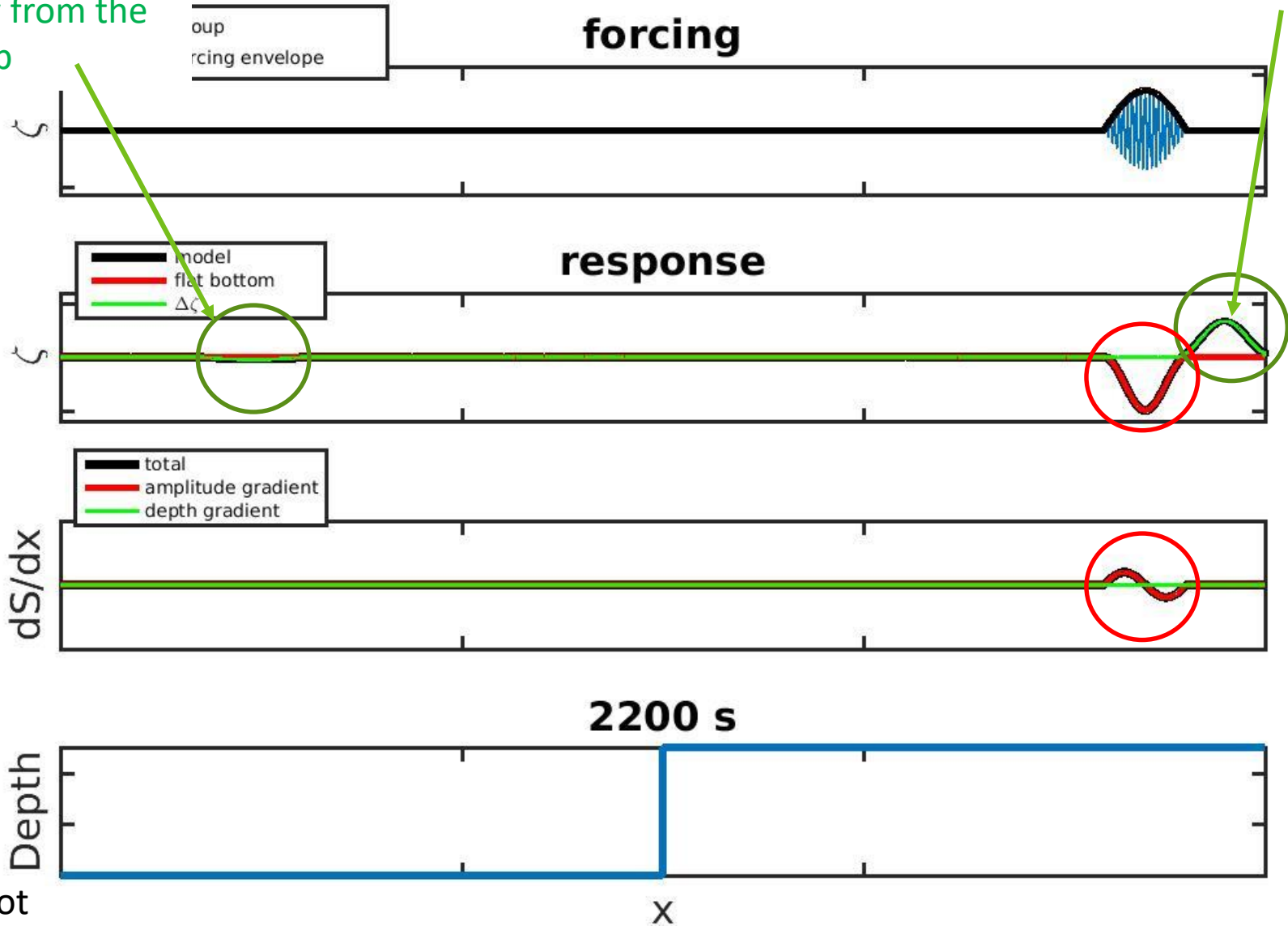


# Model run

## Single step

Outgoing Free  
Long Wave  
Small and  
propagating  
away from the  
group

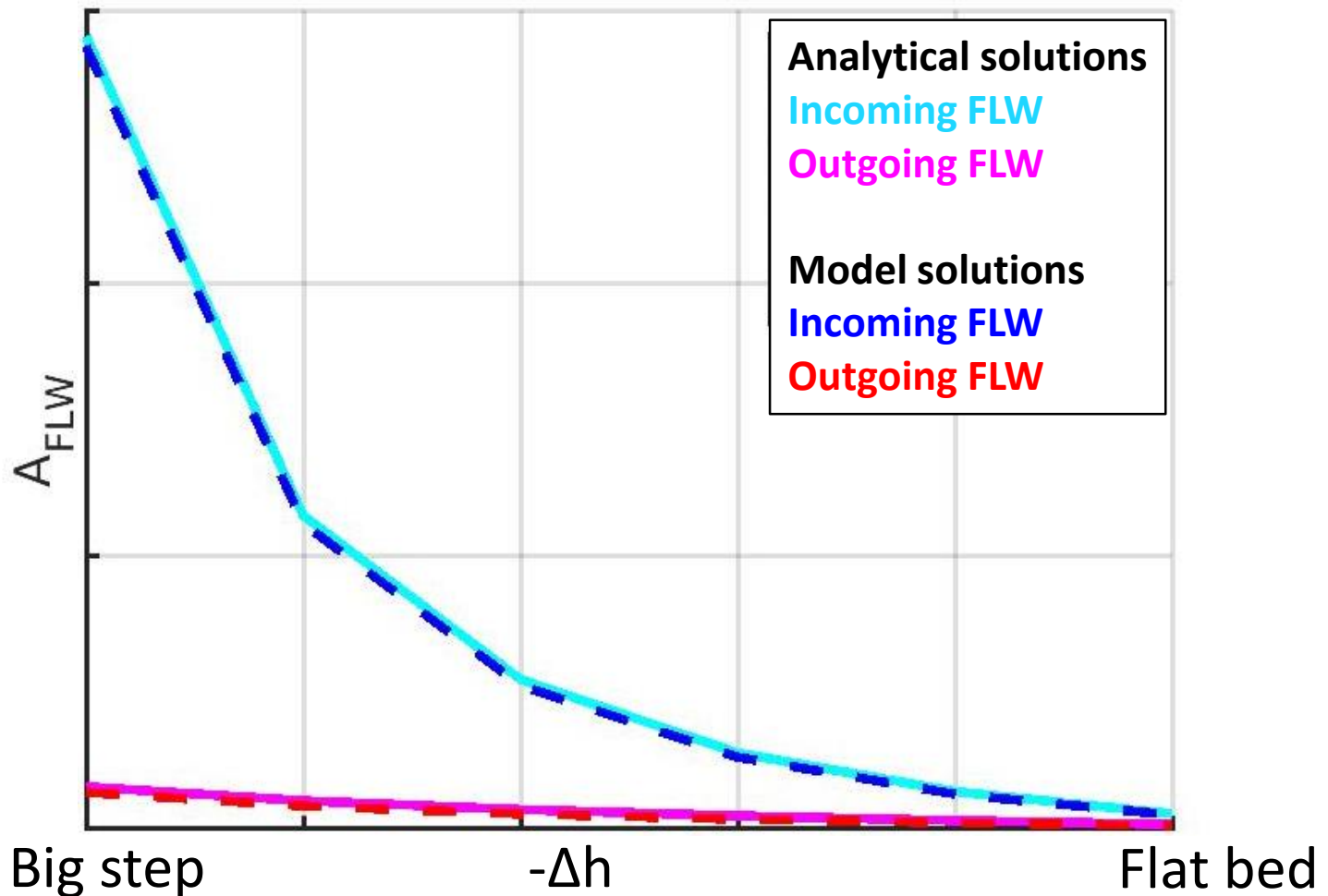
Incoming Free  
Long Wave



# Analytical solution

## Single step

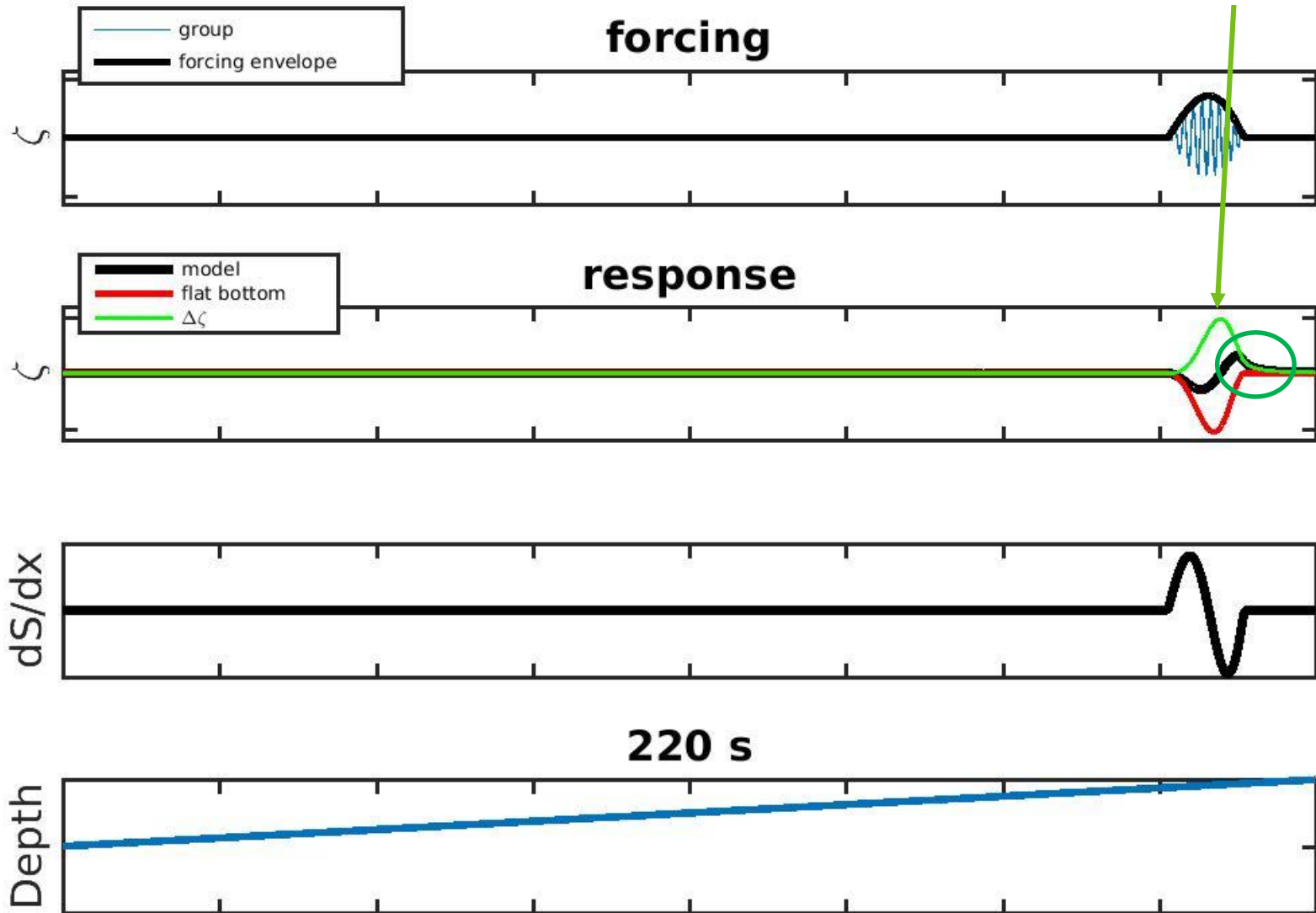
Amplitude of free long waves (FLW): incoming and outgoing



# Model run

## Back to slope

Combination of  
lagging  
Incoming Free  
Long Waves

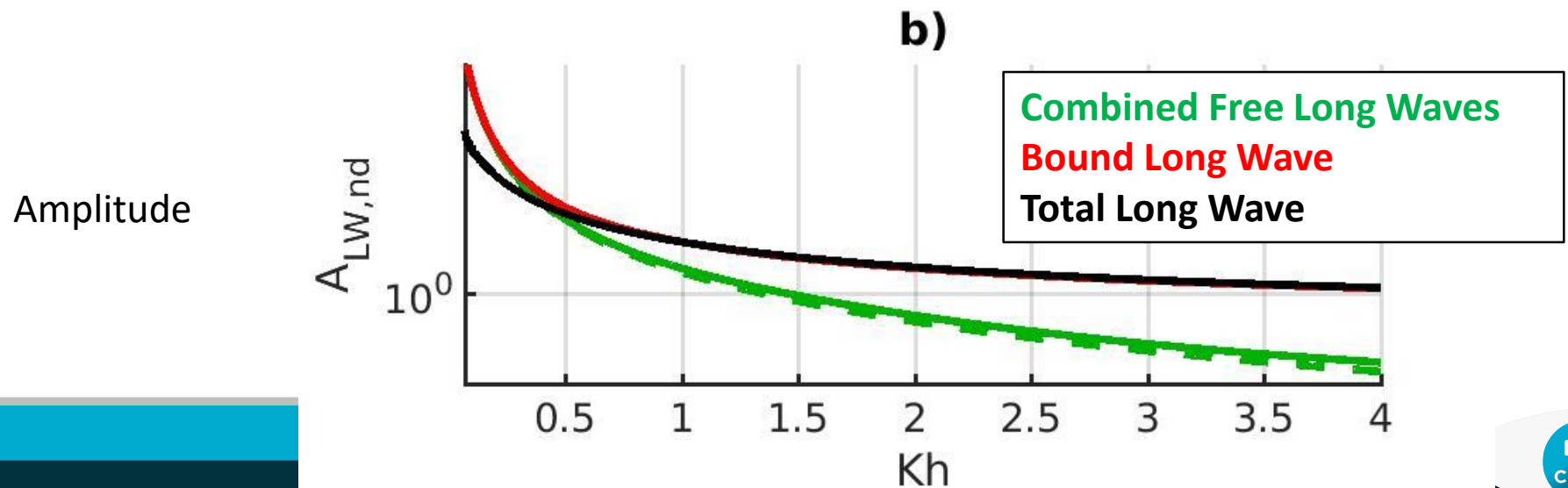
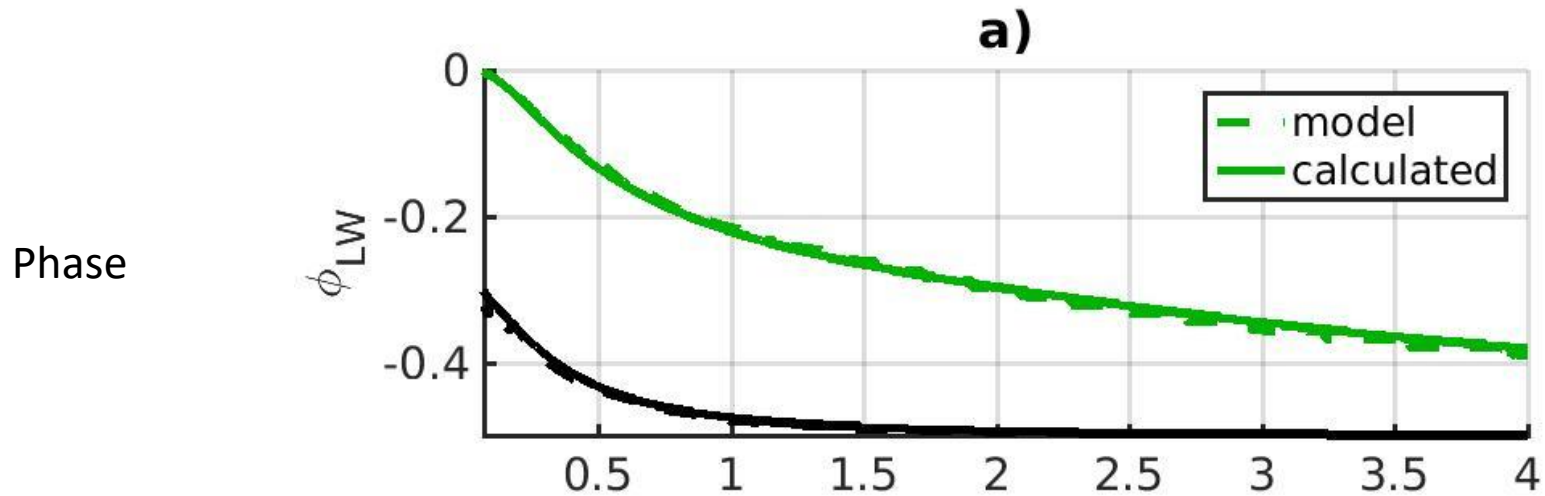


snapshot

$x$

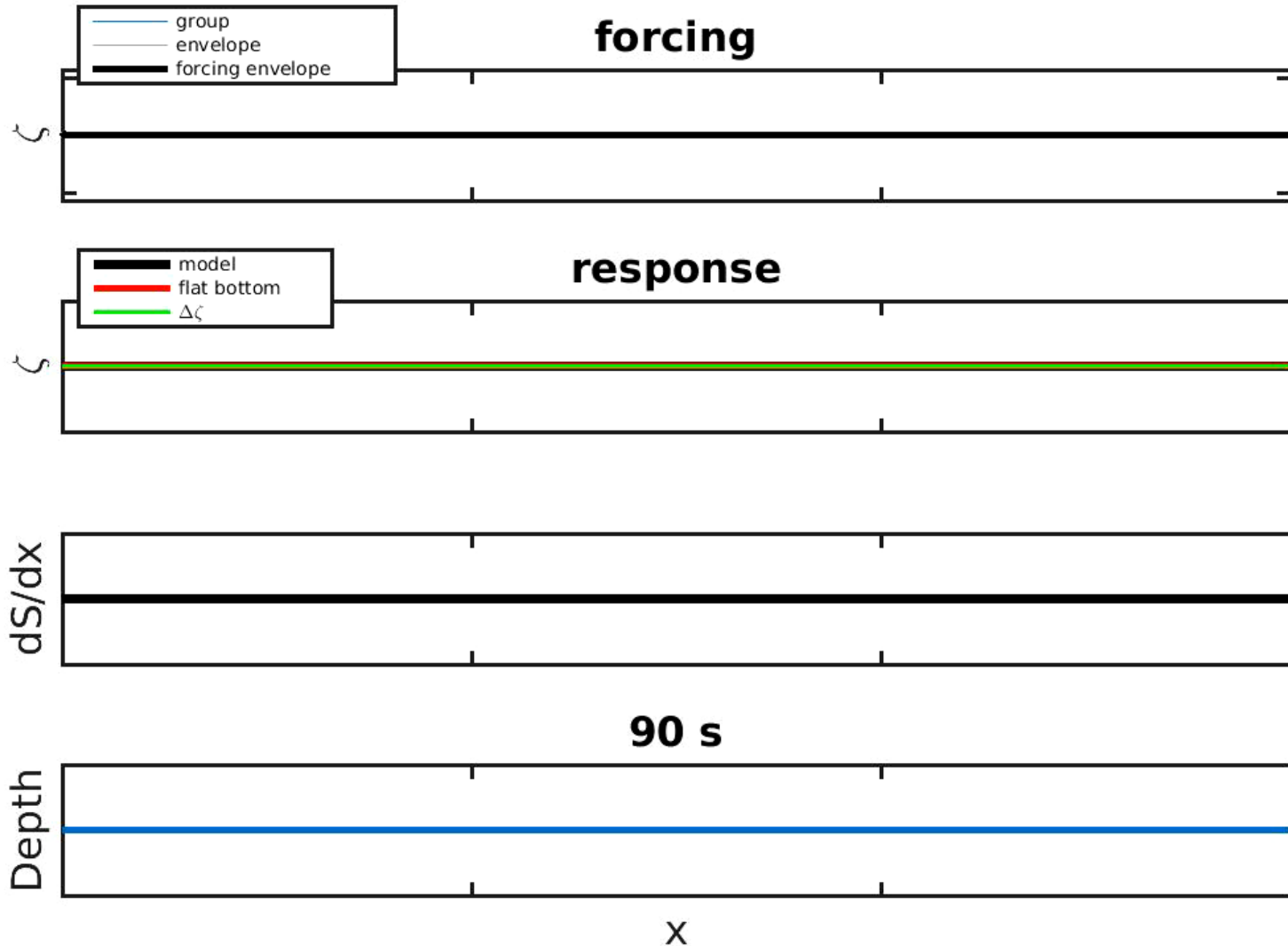
# Analytical solution

## Slope



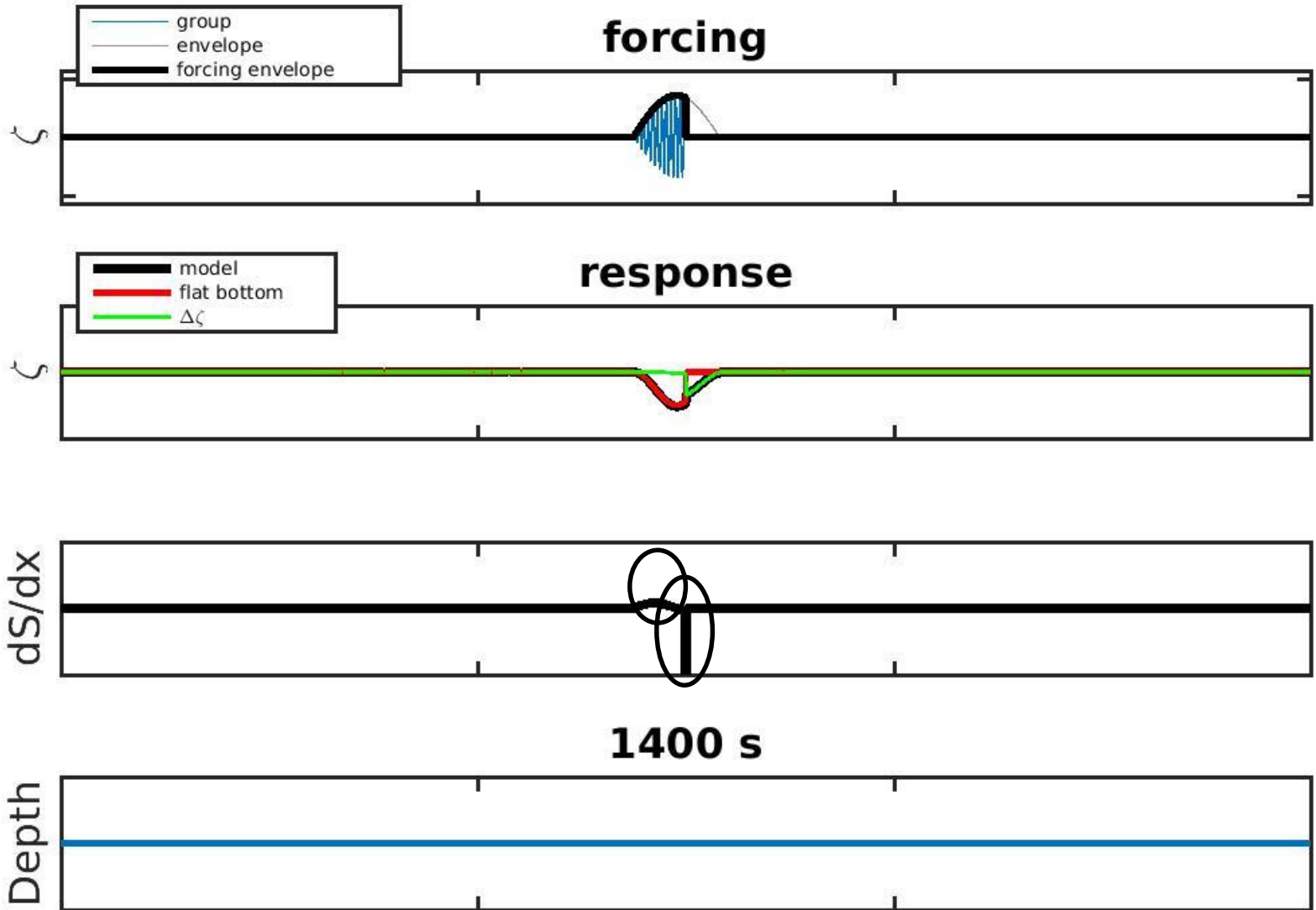
# Model run

## Flat, breaking



# Model run

## Flat, breaking

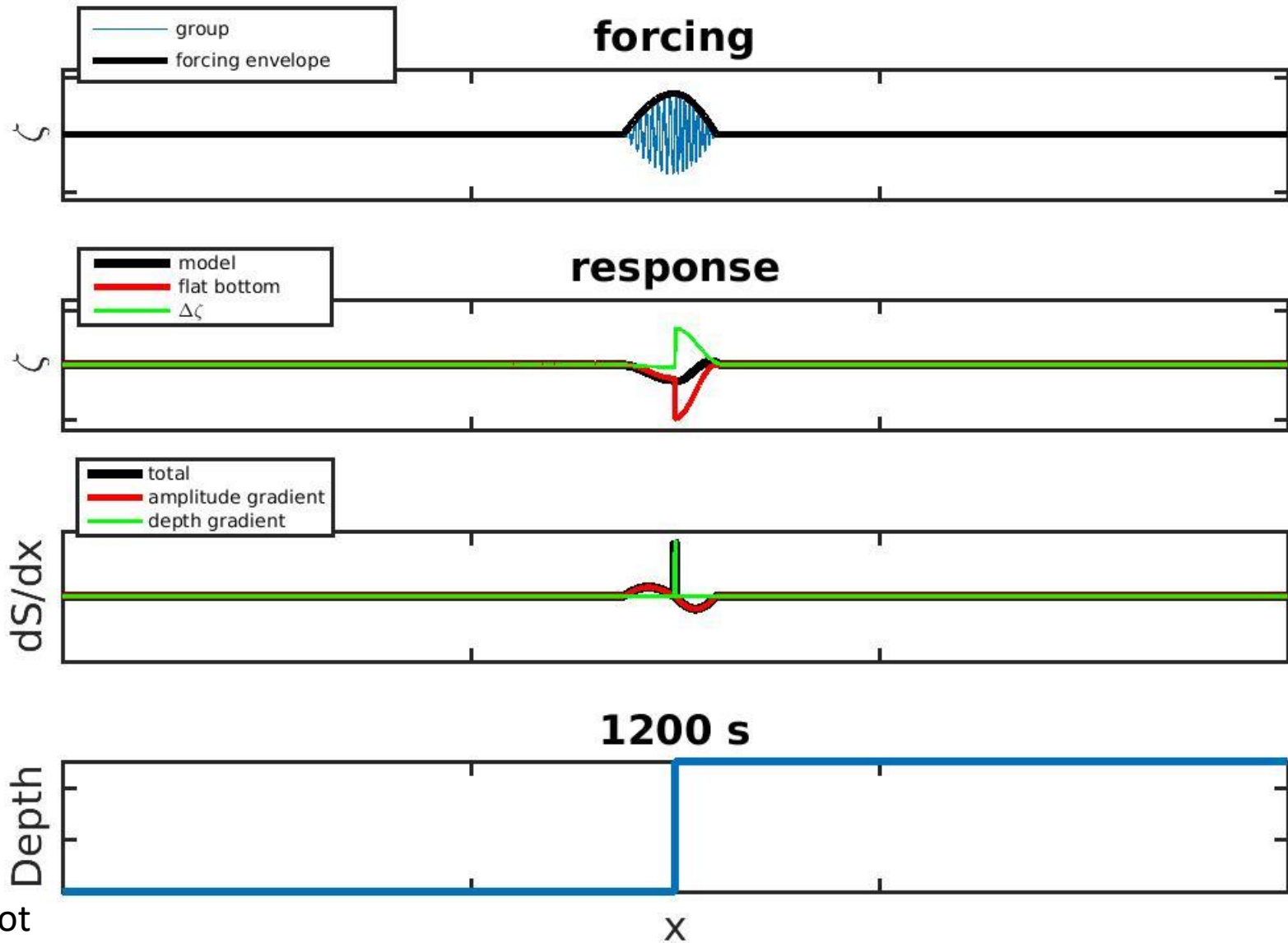


snapshot

$x$

# Model run

## Back to single step



# Conclusions

## Simple model

- Visualise the processes.
- Base to develop solutions.
- Can be forced with measured short-wave amplitude.
- Any bathymetry profile.

Solutions which explain observations.



# Conclusions

## Understanding

- Role of the radiation stress gradients.
- Radiation stress gradients caused by short-wave group amplitude gradients or depth gradients.
- Similarity between free wave generation via breakpoint forcing and depth variation.
- Co-dependence between breakpoint forcing and bound wave release

Important in the context when forecast is needed and numerical models are efficient but time consuming.

essentially the method given by Whitham (1962) when the groups are long compared to the depth. Let  $S$  denote the flux of momentum across a vertical plane and let  $S_z$  denote the difference between this and the part due to the by

$$S = \int_{-h}^{\zeta} (p + \rho u^2) dz,$$

$$S_z = \int_{-h}^{\zeta} (p + \rho u^2) dz - \int_{-h}^{\zeta} \rho g(\zeta - z) dz$$

$$= S - \frac{1}{2} \rho g (\zeta + h)^2$$

$$= S - \rho g \left( \frac{1}{2} \zeta^2 + h\zeta \right)$$

introduced by Lord Rayleigh

Cushman-Roisin

Thanks

$$\Leftrightarrow \sigma^2 \frac{\partial M}{\partial t} + \rho g \beta^2 z \frac{\partial \xi}{\partial x} = -\rho g \beta^2 z^2$$

$$\frac{\sigma^2}{\rho g \beta^2 z} \frac{\partial M}{\partial t} + \frac{\partial \xi}{\partial x} = -\frac{1}{z} \frac{\partial z^2}{\partial x}$$

$$\frac{\sigma^2 h^2}{g \beta^2 z} \frac{\partial U}{\partial t} = \frac{\sigma^2 x}{g \beta} \frac{\partial U}{\partial t} = X'$$

$$X' \frac{\partial U}{\partial t} = \frac{X'}{g \alpha \beta} \frac{\partial M}{\partial t}$$

$$\Leftrightarrow \frac{\partial \xi}{\partial t} + \frac{1}{\beta} \frac{\partial M}{\partial x} = 0$$

$$\frac{3}{2} \sigma^2 \times \beta \sigma \frac{\partial \xi}{\partial t} + \frac{1}{\beta} \frac{3}{2} \sigma^2 \times \dots$$

$$\Leftrightarrow \beta x \frac{\partial \xi}{\partial t} + \frac{1}{\beta} \frac{\partial M}{\partial x} = 0$$

$$\frac{\partial \xi}{\partial t} + \frac{1}{\beta \beta x} \frac{\partial M}{\partial x} = 0$$

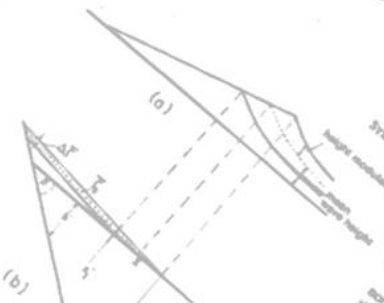


Fig. 1. Schematic representation of (a) wave height and (b) wave group. The surface is denoted by  $z = \zeta(x, t)$  and the position of the wave packet, respectively, is denoted by  $x = x_0(t)$  and  $x = x_0(t) + \Delta x$ .

where  $\sigma^2$  is the incident wave amplitude. Inside the surf zone, similarity theory indicates that the mean flow is given by  $U = \frac{1}{2} \rho g \beta^2 z^2$  where  $\beta^2$  is the beach slope (assumed constant). The surf zone is defined by the minimum and maximum positions of the wave packet, respectively, denoted by  $x = x_0(t)$  and  $x = x_0(t) + \Delta x$ . The surf zone is defined by the minimum and maximum positions of the wave packet, respectively, denoted by  $x = x_0(t)$  and  $x = x_0(t) + \Delta x$ . The surf zone is defined by the minimum and maximum positions of the wave packet, respectively, denoted by  $x = x_0(t)$  and  $x = x_0(t) + \Delta x$ .

(3)  $\frac{\partial(h, \xi)}{\partial t}$

of the incident wave. Since linearity in the group has been assumed here, the free and forced are considered separately and the solutions summed. In addition, the small angle approximation is used for the incident wave group, and the incident wave group is taken to be given by (4). We nondimensionalize (1) and (2) by scaling the variables with their mean values as follows:

$$x' = \frac{x - x_0}{\Delta x}$$

$$z' = \frac{z - \zeta_0}{\Delta z}$$

$$t' = \frac{t - t_0}{T}$$

$$U' = \frac{U - U_0}{\Delta U}$$

where  $x'$  is the mean position of the breakpoint and  $t'$  is the group frequency  $\omega T / (2\pi)$ . The scaling for  $z'$  is the mean water depth at the breakpoint, obtained by integrating (1) with  $\partial U / \partial z = 0$  from  $z' = x'$  to  $z' = 0$ . The  $U'$  scale is then found from continuity. It should be noted here that inside the surf zone we now have  $z' = x'$ . Substituting in (1) and (2) gives the following nondimensional equations:

$$\frac{\partial U'}{\partial t'} + \frac{\partial \xi'}{\partial x'} = -\frac{1}{2} \frac{\partial \xi'}{\partial x'}$$

$$\frac{\partial \xi'}{\partial t'} + \frac{\partial M'}{\partial x'} = 0$$

where  $M' = (\rho g \beta^2 / g) \sin^2 \beta$ . To solve these equations it is necessary to determine an analytic form for the forcing term on the right-hand side of (2). The forcing function can be expressed as a Fourier series as follows:

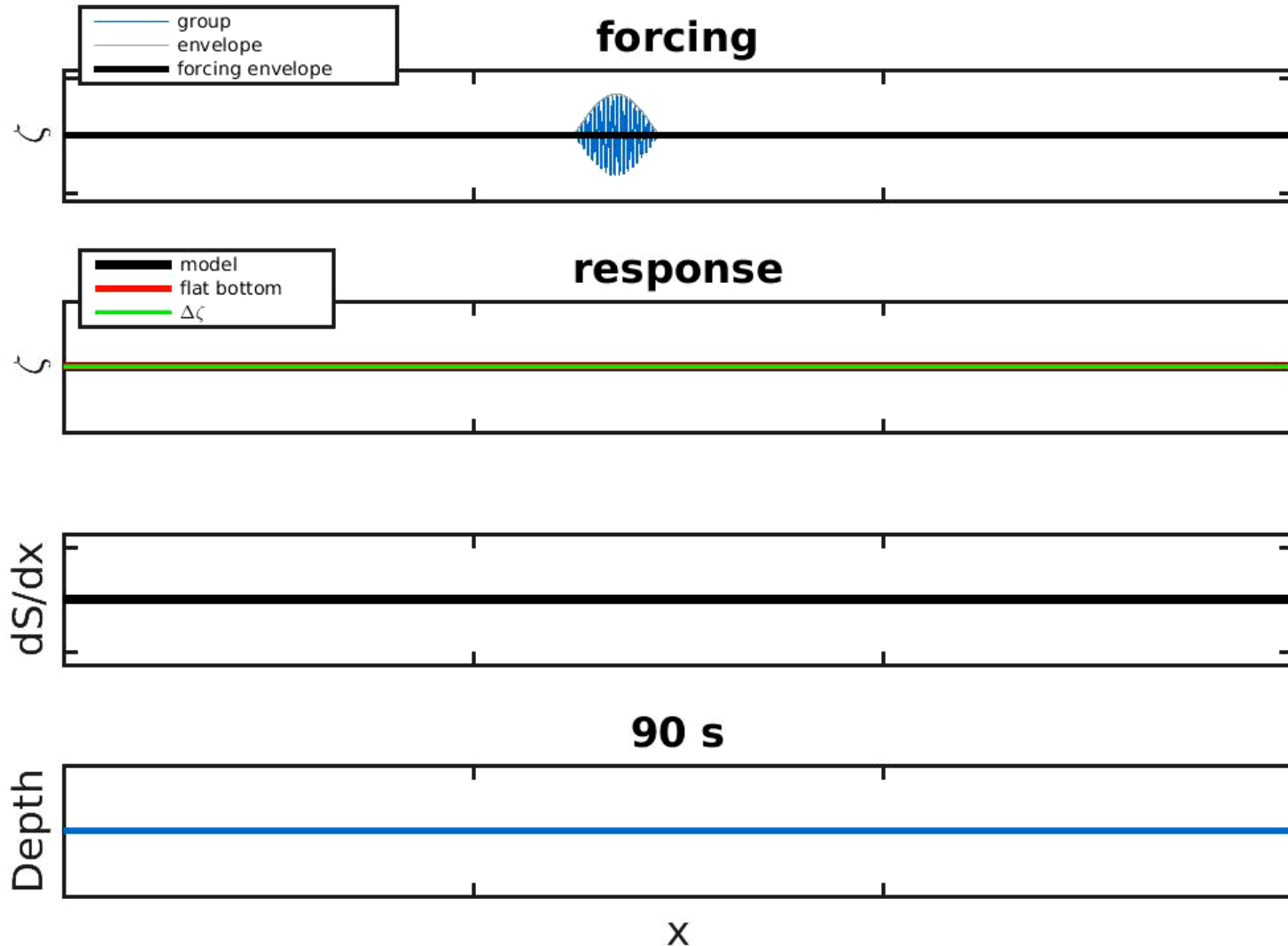
$$\frac{1}{2} \frac{\partial \xi'}{\partial x'} = \left\{ \begin{array}{l} 0 \quad x' < -x_0' \\ x' > x_0' \end{array} \right.$$

where  $x_0'$  denotes the position of the breakpoint. Equation (7) defines a constant amplitude wave whose shape is given by the function defined by (7) inside the surf zone. For  $x' < -x_0'$  and  $x' > x_0'$ , the minimum position of the breakpoint, the function is only for all time. The forcing function can be expressed as a Fourier series as follows:

$$\frac{1}{2} \frac{\partial \xi'}{\partial x'} = \frac{1}{2} \sum_{n=1}^{\infty} (a_n \cos n\pi x' + b_n \sin n\pi x')$$

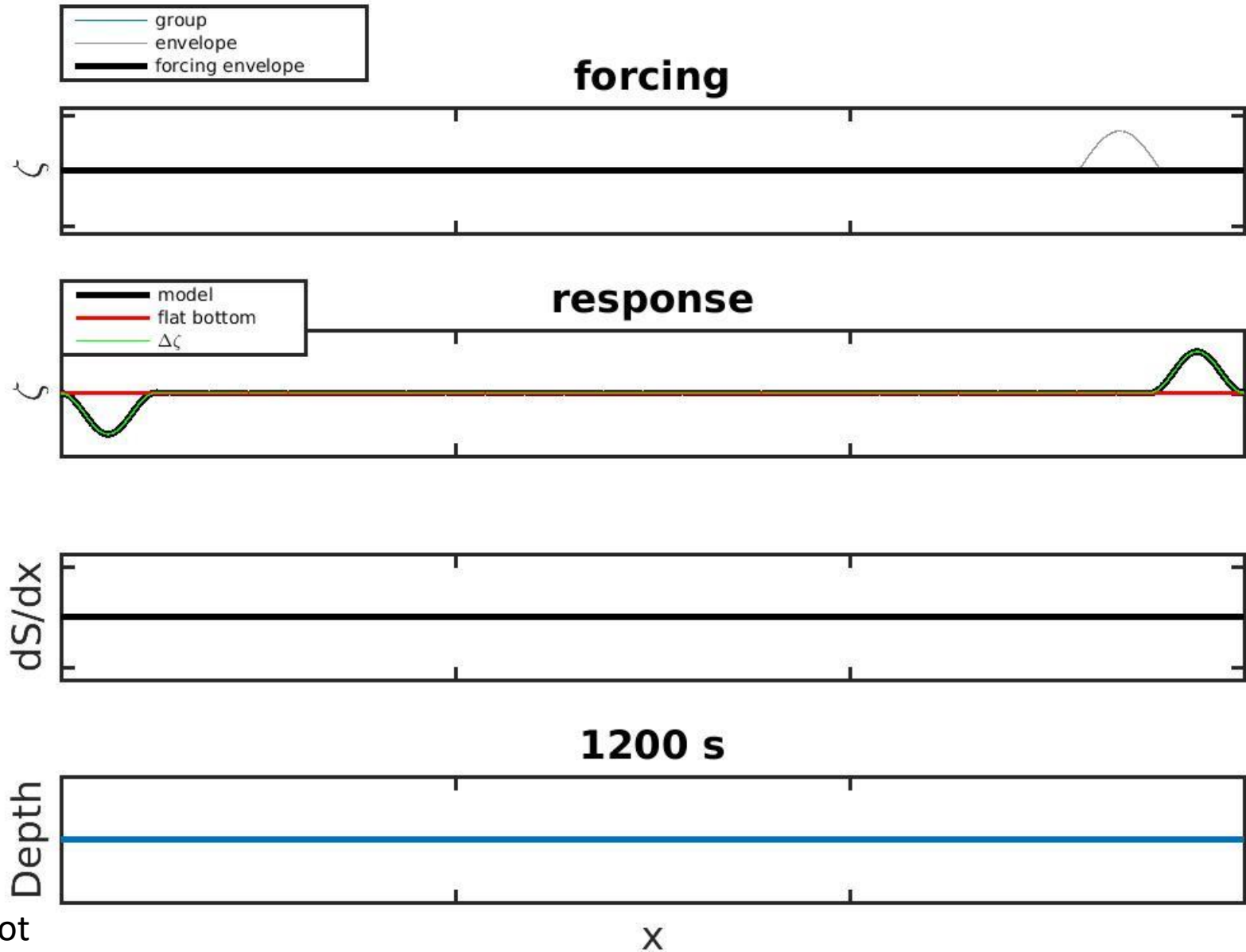
# Model run

Flat, breaking, breakpoint forcing only



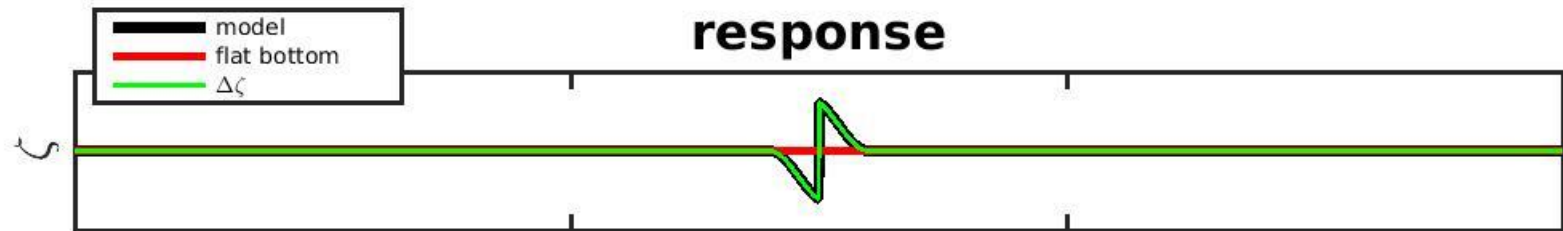
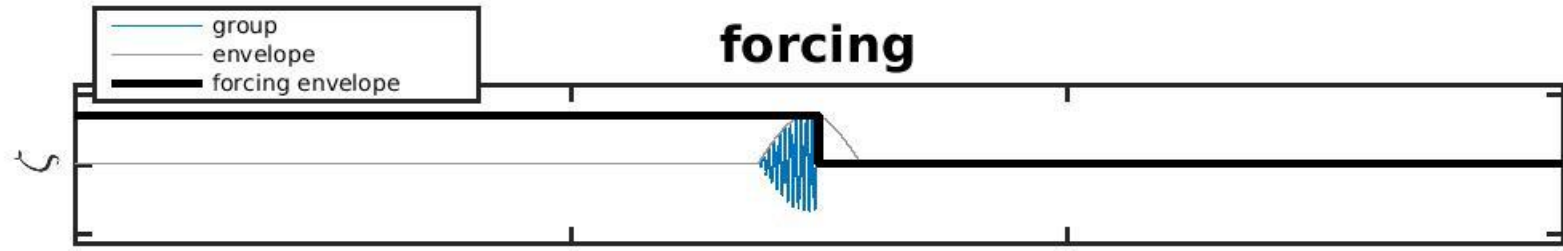
# Model run

Flat, breaking, breakpoint forcing only



# Model run

Flat, breaking, breakpoint forcing only



200 s



snapshot

$x$